A MATHEMATICAL MODEL FOR ESTIMATING THE TOTAL SOLAR IRRADIANCE FOR LAFIA, NASARAWA STATE OF NIGERIA

KUJE YOHANNA J*, ODE, S., AGANDU GONTU, J[1]., ISHAKU GDKUS, J[1], AND NENCHIN WAKTOODA, G[1].

*Department of Agricultural Engineering College of Agriculture, P M B, 33, Lafia, Nasarawa State.
[1]Department of Mechanical Engineering Plateau State Polytechnic, Barkin Ladi, Plateau State.

***corresponding author: engrkuje@yahoo.com, drkuje@gmail.com**
Article received: 11th May, 2015; Article accepted: 5th July, 2015.

ABSTRACT
An empirical mathematical model for estimating the total solar irradiance on a horizontal surface for Lafia, Nasarawa State of Nigeria (Lat 8.5°N and long. 8.5°E) was developed using the Angstrom – page equation. The model was developed from total solar irradiance, ambient air temperature, hours of bright sunshine and cloudiness measured hourly from 0600H to 1800H for 3 years for the months of January, 2009 to December, 2011. The measurements were carried out at the meteorological unit of College of Agriculture, Lafia, Nigeria. The coefficients "a" and 'b' of the Angstrom-page linear type equation were determined by plotting the clearness index ($\frac{H}{HO}$) against the fractional possible sunshine hours ($\frac{ns}{N}$) to obtain the line of best fit. The intercept of the line on the y-axis is the coefficient 'a' while the slope of this line is the coefficient 'b'. The determined model for measuring total horizontal solar irradiance at Lafia location was $\frac{H}{Ho}$ = 0.28 + 0.33 ($\frac{ns}{N}$); with a coefficient of correlation of 0.50. The mean bias difference and root mean square difference used to test the performance of 'a' and 'b' were 0.68% and 1.13% respectively. The developed model is therefore recommended to be used for estimating the total solar irradiance for Lafia location.
Keywords: Solar irradiance, mathematical model, clearness index; sunshine hours, Lafia, Nigeria.

INTRODUCTION
The knowledge of the solar resource at the earth's surface with enough accuracy is essential for planning any solar energy system at a given location. However, ground measured solar radiation is scarcely available for a given site where a solar system is planned (zarzalajo et al., 2009). The scantiness of hourly data in Nigeria makes it necessary to resort to empirical models to evaluate hourly input of diffuse (scattered) and beam (direct) irradiance for design and optimization of solar devices (Maduekwe et al., 1999; Yohanna, 2010). The comprehensive design and effective utilization of solar energy systems depends largely on adequate information of the solar radiation characteristics of the location in which the systems are located (Moradi et al., 2009, Polo et al., 2009). The best irradiance information is that obtained from experimental measurements of the total solar irradiance in question (Yohanna et al., 2011; Itodo and Yohanna, 2011).

Total radiation is the total amount of energy falling on the horizontal surface measured in J/m^2 or W/m^2. The values are usually highest in clear sky conditions and lowest during cloudy day. This total solar radiation at the ground comprises of three components namely direct (beam), diffused (scattered) and reflected radiation (Coulson, 1975; U.O.A., 1978; Pidwiring, 2005, Kastendeuch and Najjar, 2009 and Yohanna, 2010). These components are specific to a particular location, hence the need to determine the coefficients 'a' and 'b' for a particular location, if solar systems devices are to be installed at that location (Yohanna et al., 2011).

Direct radiation comes from the sun in a straight line and can cast shadows and it produces the greatest heating effect. The values are usually highest during the clear day conditions. Atmospheric (sky) radiation comes from entire sky apart from the direct sun. On a very clear day, the diffuse component amounts to 10 to 20% of the total radiation (Stine and Harrigan, 1985). The values are usually highest during the very cloudy conditions and lowest during the clear sky days (Yohanna, 2010). Reflected terrestrial radiation comes from nearby surfaces such as buildings, fences, ground it self (Howey and Bereny, 1979).

Reflected radiation is neglected incomputing total irradiance because it is difficult to be assessed (Stine and Harrigan, 1985). The contribution of the reflected radiation is of special importance in a complex environment. The probability of the radiation to be reflected before it can be trapped in a city is high (Katstendeuch and Najjar, 2009). Harman et al (2004) found an error of 18% of the total radiation when no reflection was taken into account, against only 2.5% with one reflection.

Many empirical models have been developed for different stations and meteorological conditions for the estimation of the radiation components (diffuse and direct), since stations for conducting and measuring such data are not there in many places including Lafia, Nigeria. The objective of this study therefore was geared towards developing a mathematical model of the Angstrom-Page equation from measured and computed parameters for estimating the total solar irradiance intensity for Lafia, Nasarawa State.

MATERIALS AND METHODS

Data Collection:

The experimental site was located at the meteorological unit of College of Agriculture, Lafia, Nasarawa State of Nigeria (Lat. 8.5^{o}N of the equator and long. 8.5^{o}E of the Greenwich meridian). The parameters measured hourly were solar radiation, wind speed, maximum and minimum temperatures, hours of sunshine and cloudiness and relative humidity. The parameters were measured daily between 0600H and 1800H for three years from January, 2009 to December, 2011. The averages of the 3 years were recorded, computed and used in this study.

Analysis

The Angstrom-page equation used in correlating the measure total solar irradiance (H) data with percent sunshine ($\frac{ns}{N}$) is given by the equation.

$$\frac{H}{Ho} = [a + b\,(\frac{ns}{N})\,] \text{ ------------- } (1)$$

Where H/Ho is the clearness index, H and Ho are the total terrestrial and extraterrestrial solar irradiance respectively. 'a' and 'b' are the climatic coefficients, n_s is the daily hours of bright sun shine and N is the possible daily number of hours of insolation. The ratio ($\frac{ns}{N}$) is referred to as the percent possible sunshine and the length of day, n_y is given by Lunde (1980) and Nwokoye (2006),

$$N = \frac{2}{15} \cos^{-1} (- \tan \varphi \tan \delta) \text{ -----------} (2)$$

RESULTS AND DISCUSSION

RESULTS

Where φ and δ are the latitude and declination angles respectively. The declination angle δ, is given by equation 3 (Anderson, 1983).

$$\delta = 23.45 \sin [360(284 + \frac{ny}{365})] \text{ --------- } (3)$$

Where n_y – mean day of each of the year given by Lunde (1980).

The extraterrestrial radiation (Ho) is given by equation 4(Stine and Harrigan, 1985)

$$Ho = \frac{36400}{\pi} Isc\,[1 + 0.034\,(\frac{360ny}{365})]\,[\cos \varphi \cos \delta \sin \cos + \frac{\pi}{180}\,\omega_s \cos \sin \varphi \sin \delta] \quad J/m^2 \text{ or } W/m^2 \text{ ------------------ } (4).$$

1sc-solar constant = 1367 Wm^2, in SI unit is 4921 $KJ/hr/m^2$ (Stine and Harrigan, 1985), Page, 1986; Nwokoye, 2006; Montero at al., 2008).

ω_s – Sun set hour angle for a typical day n, of each month in degrees = $\cos^{-1} (- \tan \varphi \tan \delta)$

φ – Latitude angle of the location, degrees

δ - Declination angle for the month, degrees.

The climatic coefficients 'a' and 'b' were determined by plotting the clearness index (H/Ho) on the y-axis and the fractional possible sunshine hours (ns/N) on the x –axis to obtain the line of best fit. The intercept of this line on the y-axis is the diffuse (scattered) radiation 'a' while the slope of the line is the beam (direct) radiation 'b'.

The mean bias difference (MBD) and root mean square difference (RMSD) are given by equations 5 and 6 respectively (Maduekwe and Chendo, 1992, Akingbade, 1999; Maduekwe et al, 1999).

$$MBD\,(\%) = [\,\sum \frac{(Hest - Hmea)}{n}]/[\,\sum \frac{Hmea}{n}] \text{ -----} (5)$$

$$RMSD\,(\%) = [\sum(\frac{Hest\text{-}Hmea}{n})^2]^{1/2} / [\sum(\frac{Hmea}{n})] \text{ -------} (6)$$

Where Hest – estimated or predicted hourly clear sky solar irradiance (or horizontal irradiance).

Hmea – measured or observed hourly clear sky (or horizontal global luminance).

n – number of data points (months) or population size.

Table 1. Summary of monthly average daily total solar irradiance intensity parameters

Month/ years	**H(W/m^2)**	**H(w/m^2)**	$(\frac{H}{H_o})$	**ns (hr)**	**Ns (hr)**	**(ns/N)**	**Hpred (W/m^2)**
Jan	228.05	386.15	0.59	8.94	11.64	0.77	228.90
Fed	257.70	410.23	0.63	9.43	11.77	0.80	257.92
Mar	263.80	430.87	0.61	9.84	11.96	0.82	262.57
April	242.75	434.85	0.56	9.43	12.16	0.78	241.57
May	236.55	425	0.56	8.87	12.32	0.72	237.58
Jun	221.05	416.08	0.53	8.23	12.41	0.66	225.68
Jul	197.35	418.76	0.47	6.87	12.37	0.56	198.36
Aug	167.85	428.06	0.39	6.48	12.23	0.53	170.73
Sep	217.13	429.72	0.51	8.10	11.83	0.68	213.32
Nov	187.62	390.17	0.48	9.70	11.67	0.38	189.65
Dec	164.46	376.71	0.44	9.71	11.59	0.85	167.57
Total	2808.77	4960.82	6.28	103.2	143.98	8.62	2616.04
Average	234.06	413.40	0.52	8.60	12.00	0.72	218.00

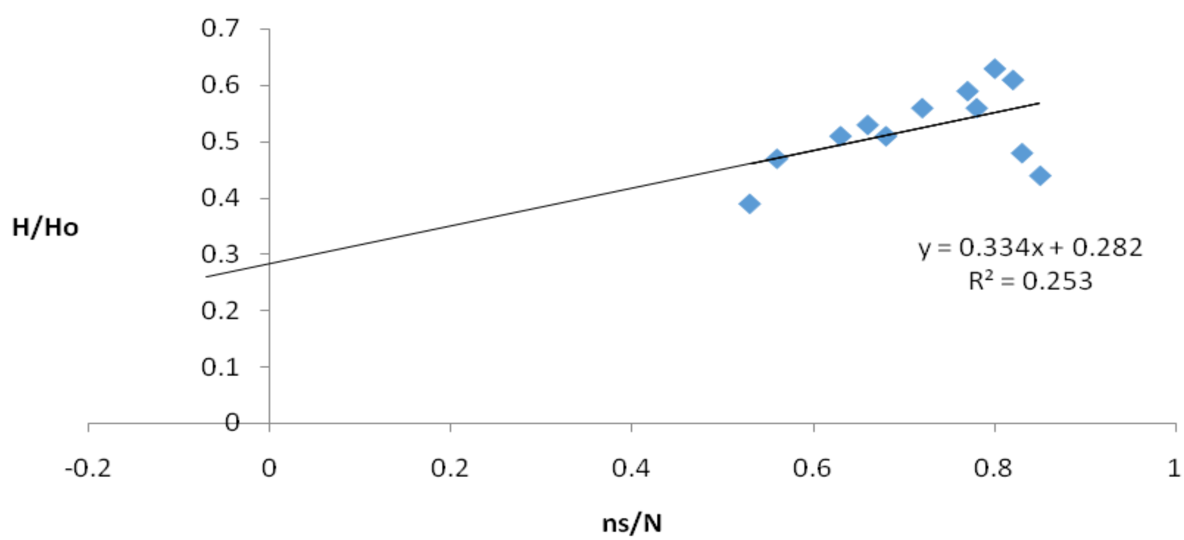

Fig .1. Plot of clearness index (H/Ho) versus fraction possible sunshine hours (ns/N) for Lafia location based on monthly means.

DISCUSSION OF RESULTS

Table 1 is the summary of the average daily global solar irradiance parameters for Lafia, Nasarawa state of Nigeria. Fig 1 is a graphical plot of the clearness index (H/Ho) versus the fractional possible sunshine hours (ns/N) for Lafia location based on monthly means. The value of the diffuse (or scattered) irradiance (a) was 0.28 while the beam (or direct) irradiance 'b' was 0.33 with correlation coefficient of 0.50. The determined Angstrom – page equation for predicting the monthly average daily total solar irradiance for Lafia, Nasarawa state of Nigeria, is H/Ho = 0.28 + 0.33 (ns/N). The coefficients 'a' = 0.28 and b=0.33 compared favourably with that of Akinbode (1992) with a=0.246 and b=0.4296 for Minna. The coefficient of correlation for the prediction equation showed that 50% of the variations in the measured solar intensity parameters were explained by the developed model leaving 50% to be explained by other factors, which may change with time and location

The mean bias difference (MBD) and the root square differences (RMSD) were used to validate the performance of the model. The MBD and RMSD for the determined equation were 0.68% and 1.13% respectively. The very low value of MBD than RMSD indicates that the average amount over estimation in the calculated values is nearly equal to the amount of under estimation hence the more accurate of the developed model for this study as stated in Maduekwe and Chendo (1992).

CONCLUSION

Based on the results obtained, it is concluded that:

i. The coefficients 'a' and 'b' for Lafia, Nigeria are 0.28 and 0.33 respectively.
ii. The total solar irradiance for Lafia location can be estimated from the equation, H/Ho = 0.28 + 0.33 (ns/N).
iii. The developed mathematical model can therefore be used to estimate the total solar irradiance for Lafia and its environs.

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