NSUK Journal of Science & Technology, Vol. 7(1): pp. 4-6, 2020 Print: ISSN: 2756-696X
On-line: ISSN 2756-6951
Website: www.nsukjst.org.ng

GRAVITOELECTRIC COUPLING IN A SLOW MOTION AND WEAK RELATIVISTIC FIELD

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ABSTRACT
The formal idea of gravitoelectric component has a long history, the gravitoelectric component springs from the weak field and slow motion approximation of Einstein geometrical field equations of gravity and by a common analogy with Maxwells dynamical field equations of electromagnetism. The gravitoelectric component is known to account for the excess perihelion precession of the orbit of mercury as a relativistic correction to the Newtonian idea. In this article, we study the non-static gravitoelectric component using a weak relativistic field and slow-motion approximation in an Einstein-Maxwellian type equation to establish the coupling effect between non static gravitational field and electric field. The proposed study contains the non-static Einstein's weak field gravitational scalar potential applied to d'Alembertian operator to obtain the Poissons equation. The study satisfies Guage transformation, the continuity of functions and shows unqiue method to obtain the scalar potential and vector potential of gravitoelectric component by seeking some series approximations and boundary condition to the gravitoelectric field. Our obtain result in the limit of weak field establishes a coupling effect between Einstein geometrical field equations and Maxwells dynamical field equations, it also satisfies Poisson'ss equation and the verified equivalence principle of Physics. The result will widen the scope for further study and laboratory experimentation of gravitoelectric coupling.

***Keywords**: Gravitoelectric component, Coupling effect, Laplacian operator, Relativistic field, Weak field.*

INTRODUCTION

The concept of gravitoelectromagnetism in the limit of weak field has recently attracted a lot of attention and has been a subject of intense research in Ref. (Flavia & Rubens 2017; Sarki *et al*., 2019; Bezerra *et al*., 2005; Nyambuya, 2014; Tajmar, 2001).

Einstein in 1916 proposed the theory of General Relativity (GR), which describes how space is affected by mass-energy and momentum tensor distribution (Flavia & Rubens 2017; Sarki *et al*., 2019). Linearization in the weak field and slow-motion approximation has unveiled in previous literature a set of four (4) non-linear differential equations comparable to Maxwell's equation of electromagnetism (Flavia & Rubens 2017; Nyambuya, 2014; Tajmar, 2001; Howusu, 2009, 2010a, 2010b). The effects of GR can be better understood by using a direct formal analogy with electromagnetism, and the idea is that non-static moving mass currents generate a field (Bezerra *et al*., 2005). Einstein's GR provided an excellent explanation of the excess motion of Mercury's perihelion in terms of a relativistic gravitoelectric correction to the Newtonian gravitation.

Subsequently in 1918, Lense & Thirring discovered the Gravitomagnetic effect of the Einstein's field equation in a slow-motion and weak-field approximation, the consequence of this is that Einstein's Geometrical Field Equation EGFE could be written in the same structure as Maxwell's Dynamical field equations of Electromagnetism MDTE with the existence of comparable operators for both field (Sarki *et al*., 2019). Thus, the formalism of gravitoelectromagnetic coupling arises from the interaction and splitting of electromagnetic into components.

According Maxwell's equations a time-varying (non-static) magnetic field $\vec{B}$ acts as a source of electric field $\vec{E}$. Thus either a time-varying magnetic field $\vec{B}$ or electric field $\vec{E}$ induces a field of the order in adjacent regions which can propagate through space with speed equals to light . Maxwell's equation for time varying field is proposed to be Eqn. (1-4) (Tajmar, 2001; Young & Freedman, 2008; Tiwari & Malav 2010).

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \tag{1}$$

$$\nabla \times \vec{H} = J + \frac{\partial \vec{D}}{\partial t} \tag{2}$$

$$\nabla \bullet \vec{B} = 0 \tag{3}$$

$$\nabla \bullet \vec{D} = \rho_v \tag{4}$$

Where ρ_v is the charge density

While the Einstein gravtoelectromagnetic analog of Maxwells equation is proposed to be (5)- (6)

$$\nabla \times \vec{g} = -\frac{\partial \vec{B}_g}{\partial t} \tag{5}$$

$$\nabla \times \vec{H}_g = -\mu_0 J + \frac{1}{c^2}\frac{\partial \vec{g}}{\partial t} \tag{6}$$

$$\nabla \bullet \vec{g} = -\frac{\rho_v}{\varepsilon_g} \tag{7}$$

$$\nabla \bullet \vec{B}_g = 0 \tag{8}$$

Where $\vec{g}$ is the gravitoelectric field, $\vec{B}_g$ is the gravitomagnetic and ε_g is the gravitational permittivity.

In this article we establish theoretically the coupling effect between EGFE and MDTE, and using a weak field approximation to gravitectric potential

THEORETICAL BACKGROUND

Metric tensor are the fundamental basis for all geometrical theories and the metric tensors should hence satisfies Scwarszchild metric and should naturally reduce to a second order partial differential equation (Sarki *et al*., 2019; Howusu, 2010a, 2010b). The metric tensor to a non-static varying distribution is given by Chifu (2009) as

$$g_{00} = 1 + \frac{2f(t,r)}{c^2} \qquad (9)$$

$$g_{22} = \left(1 + \frac{2f(t,r)}{c^2}\right)^{-1} \qquad (10)$$

$$g_{33} = -r^2 \qquad (11)$$

$$g_{00} = -r^2 \sin^2\theta \qquad (12)$$

$$g_{\mu\nu} = 0 \qquad (13)$$

It could be seen that the Eqn. (9)-(13) satisifes the priori and according to Heaveside the gravitoelectromagnetic field are hidden in Einstein tensor equation and can be written as a linear perturbation of the Minkwoski spacetime. the geometrical wave equation in (9)-(13) in the limit of weak gravitational field and slow motion reduces to the wellknown d'Alembertian operator given by Chifu (2009) as

$$\nabla^2 f(t,r) + \frac{1}{c^2}\partial_t^2(t,r) = 0 \qquad (14)$$

For a non static gravitoelectric field Eqn (7), the gravitoelectric field will be express as the scalar potential of the gravitoelectric field

$$\vec{g}(t,r) = -\nabla\Phi_g \qquad (15)$$

It is established that the divergence of a curl is zero, and by computation from Amperes law it could be shown in (Nyambuya, 2014, Howusu 2010a, 2010b). that

$$\nabla^2\Phi_g(t,r) + \frac{1}{c^2}\partial_t^2(t,r) = -\frac{\rho_v}{\varepsilon_g} \qquad (16)$$

However towards the interior Eqn. (16) = 0

Hence at this stage coupling effect is well established as given in Eqn. (14) and (16). The solution to the gravitoelectric field potential is eminent.

For a relativistic non-static field, Eqn. (14)

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left[r^2\left(1+\frac{2}{c^2}f(t,r)\right)\frac{\partial\vec{g}(t,r)}{\partial r}\right] - \frac{1}{c^2}\frac{\partial}{\partial t}\left(1+\frac{2}{c^2}f(t,r)\right)\frac{\partial\vec{g}(t,r)}{\partial t} = -\frac{\rho_v}{\varepsilon_g} \qquad (17)$$

And in a recent article the scalar potential is (Sarki *et al*., 2019; Lumbi, *et al*., 2019).

$$f(t,r) = \frac{k}{r}\left[1 + \frac{\omega^4}{8}\left(t^4 - \frac{4t^3 r}{c^2} + \frac{6t^2 r^2}{c^2}\right)\right] \qquad (18)$$

Taking the derivative of Eqn (16) with respect to r gives

$$f^I(t,r) = -\frac{k}{r}\left[1 + \frac{\omega^4}{8}\left(t^4 - \frac{4t^3 r}{c^2} + \frac{6t^2 r^2}{c^2}\right)\right] + \frac{k\omega^4}{8r}\left[\left(\frac{12t^2 r}{c^2} - \frac{4t^3}{c}\right)\right]$$

(19)

Substitute Eqn. (18) and (19) into Eqn. (17) while taking the derivative to the weak field. in the limit of c^o

The equation reduces to the Poisons form, giving by

$$\frac{2}{r}\vec{g}(t,r) + \vec{g}^I(t,r) = -\frac{\rho_v}{\varepsilon_g} \qquad (20)$$

While in the limit of c^{-2} ,

$$\left(\frac{-2k}{c^2r^2} - \frac{k\omega^4t^4}{4c^2r}\right)\vec{g}(t,r) + \left(\frac{2}{r} + \frac{4k}{c^2r^2} - \frac{k\omega^4t^4}{4c^2r}\right)\vec{g}^I(t,r) + \left(1 + \frac{2k}{c^2r} - \frac{k\omega^4t^4}{4c^2r}\right)\vec{g}^{II}(t,r) - \frac{1}{c^2}\vec{g}(t,r) = -\frac{\rho_v}{\varepsilon_g}$$

(21)

For this exterior field equation, we employ the weak field approximation to Eqn. (21) In order to obtain the vector potential by .

The series solution for the exterior scalar potentialcan be written in the form

$$\vec{g}^+(t,r) = \frac{\vec{g}_1}{r} + \frac{\vec{g}_2}{r^2} + ... \qquad (22)$$

$$\vec{g}^{I+}(t,r) = -\frac{\vec{g}_1}{r^2} - \frac{2\vec{g}_2}{r^3} + ... \qquad (23)$$

$$\vec{g}^{II+}(t,r) = \frac{2\vec{g}_1}{r^3} - \frac{6\vec{g}_2}{r^2} + ... \qquad (24)$$

Substitute the series terms in Eqn (22)-(24) to the weak field, in Eqn. (21) gives

$$\left(\frac{-2k}{c^2r^2} - \frac{k\omega^4t^4}{4c^2r}\right)\left(\frac{\vec{g}_{1,}}{r} + \frac{\vec{g}_2}{r^2}\right) + \left(\frac{2}{r} + \frac{4k}{c^2r^2} - \frac{k\omega^4t^4}{4c^2r^2}\right)\left(-\frac{\vec{g}_{1,}}{r^2} - \frac{2\vec{g}_2}{r^3}\right)$$
$$+\left(1 + \frac{2k}{c^2r} - \frac{k\omega^4t^4}{4c^2r^2}\right)\left(\frac{2\vec{g}_{1,}}{r^3} - \frac{6\vec{g}_2}{r^2}\right) - \frac{1}{c^2}\left(\frac{\vec{g}_{1,}}{r} + \frac{\vec{g}_2}{r^2}\right) = -\frac{\rho_v}{\varepsilon_g}$$

(25)

Camparing the coeffiecient of $\frac{1}{r}, \frac{1}{r^2}, \frac{1}{r^3}$ with the condition of linear dependency of r on $\vec{g}$, in which we obtain expression for $\vec{g}_{1,}\vec{g}_2$

$$g_2 = \frac{8\vec{g}_1}{\omega^4 t^4}, \qquad (26)$$

$$\vec{g}_1 = \frac{\omega^4 t^4 \vec{g}_2}{8} \qquad (27)$$

$$\vec{g}(t,r) = \frac{\vec{g}_1}{r}\left(1 - \frac{8}{\omega^4 t^4 r}\right) + ... \qquad (28)$$

Adopting the guage transformation, gravitoelectrtic scalar potential satisfies the condition of continuity of a function, the contiuity functions thus is given by

$$\left(\frac{\partial \vec{g}^{+}}{\partial r}\right)_{r=R} = \left(\frac{\partial \vec{g}^{-}}{\partial r}\right)_{r=R} \qquad (29)$$

which invariable will be given by $2D_2R = -\frac{\vec{g}_1}{R^2} - \frac{2\vec{g}_2}{R^3}$ (30)

substituting Eqn (26) to Eqn (30) yields

$$-\frac{3\rho_v \vec{g}}{2\pi\varepsilon_0 R^3} = \frac{\vec{g}_1}{R^2} + \frac{16\vec{g}_1}{\omega^4 t^4 R^3} \qquad (31)$$

We can write Eqn. (20) interms of $\vec{g}_1$ and substitute in Eqn. (28), which yields

$$\vec{g}^{+} = -\frac{3\rho_v \vec{g}}{2\pi\varepsilon_0 R^3}\left(\frac{1}{2r} + \frac{4}{\omega^4 t^4} + 1\right)\left(\frac{16}{\omega^4 t^4}\right)^{-1} \qquad (32)$$

We can similarly obtain the equation for the interior field with the series term $\vec{g}_1^{\,-} = D_2 r^2 + D_4 r^2 + ...$

Which reduces the interior field to a pure Poisons-Maxwells equation.

Thus, Equation (32) is the total gravitoelectric potential energy due to the exterior field.

The gravitoelectric potential using Einstein geometrical field equations in the limit of weak relativistic field is indeed a profound theoretical discovery, as of recent potentials have become powerful tools in quantum physics in the study of non-linear systems.

Our obtain potential could be apply in the Schroudinger equation to study energy spectrum, charge particles and wave function properties in the presence of gravitoelectric field.

This potential has an analogy with Coulomb potential given by $\mathrm{V(r)} = \frac{\mathrm{Ze}^2}{4\pi\varepsilon_0 r}$ and The trigonometric Pöschl-Teller potential used in describing diatomic molecular vibration given by $V(r) = V_1 \cos ec^2(\alpha r) + V_2 \sec^2(\alpha r)$

CONCLUSION

Thus, in this article we have seen the interaction between non-static Einsteins relativistic geometrical field equation and Maxwell's dynamical field equations, thus the coupling effect is well established and our study satisfies Poisons equations given by (20) thus equivalence principle of Physics is well established.

The gravitoelctric coupling has establsihed a relationship between Einstein geometrical theory of relativity and Maxwells dynamical theory of electromagnetism, this is indeed an encouraging factor for further laboratory experimentation of gravitoelectric coupling.

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AKNOWLEDGEMENT

The authors wishes to aknowledge the anonymous reviewers for their comments which generally improve the content of the manuscript and to late Professor S.X.K. Howusu for the provision of various litereatures on geometrical theory.