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INSTRUMENT FOR MODELING OPTIMIZATION PROBLEMS-SOLVING PROCESS: IMPLICATIONS FOR MATHEMATICAL EXPRESSION

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ABSTRACT

This paper investigated the instrument for modeling optimization problems-solving process with implications for mathematical expression. The paper further highlighted the instrument for modeling optimization problems-solving process, classification of models, typologies of mathematics modeling, steps in the optimization problem-solving process, reasons of using mathematical model and methods of teaching mathematical optimization with the implications. The implications of mathematical expression of optimization problem-solving consists of a set of variables that describe the state of the system, constraints that determine the states that are allowable, external input parameters and data, objective function that provides an assessment of how well the system is functioning. The important step in the optimization process is classifying as optimization model. Models with discrete variables are discrete optimization problems solving process, models with continuous variables are continuous optimization problems solving process. This paper concludes that mathematical models are the best way of solving mathematical problem in terms of teaching and learning of mathematics.

Keywords: *Instrument, modeling, optimization problems and implication.*

INTRODUCTION

Instrumentation is a collective term for measuring instruments used for indicating, measuring and recording physical quantities, and has its origins in the arts, social science and science of scientific instrument-making. The term instrumentation may refer to a device or group of devices used for direct reading thermometers or, when using many sensors, may become part of a complex industrial control system in such as manufacturing industry, vehicles and transportation (Muthauf, 1961). Instrumentation research is employed for mathematical construction if it is used for measuring and evaluating psychological traits, it therefore follows logically that without instrumentation research, human abilities or social and psychological constructs cannot be satisfactorily measured and evaluated. This shows how important and indispensable an overview of instrument research is instrumentation can be found in the household as well; a smoke detector or a heating thermostat is an example.

A mathematical instrument it has to do with a tool or device used in the study or practice of mathematics (Ted, 2012). In mathematical geometry, construction of various proofs was done using only a compass and straightedge; arguments in these proofs relied only on idealized properties of these instruments and literal construction was regarded as only an approximation. In applied mathematics, mathematical instruments were used for measuring angles and distances, in astronomy, navigation, surveying and in the measurement of time. Instruments such as the astrolabe, the quadrant, and others were used to measure and accurately record the relative positions and movements of planets and other celestial objects. An optimal instruments calibration technique based on the method of least squares and on correction with comparable argument and function errors is described. Expressions are derived for calculating linear regression coefficients and calibration errors.

A model is a representation of a system that allows for investigation of the properties of the system and, in some cases, prediction of future outcomes. Model is often used in quantitative analysis and technical analysis, and sometimes also used in fundamental analysis. A model is a theoretical construct representing economic processes by a set of variables and a set of logical and/or quantitative relationships between them. A mathematical model is a description of a system using mathematical concepts and language. Mathematical model is used in the natural sciences and engineering disciplines, as well as in the social sciences. The process of developing a mathematical model is termed mathematical modeling. Mathematical modeling is a principled activity that has both principles behind it and methods that can be successfully applied. The principles are over-arching or metal principles phrased as questions about the intentions and purposes of mathematical modeling.

According to Hossein (2015) mathematical optimization is the branch of computational science that seeks to answer the question `What is best?' for problems in which the quality of any answer can be expressed as a numerical value. Such problems arise in all areas of business, physical, chemical and biological sciences, engineering, architecture, economics, and management. The range of techniques available to solve them is nearly as wide. A mathematical optimization model consists of an objective function and a set of constraints expressed in the form of a system of equations or inequalities. Optimization models are used extensively in almost all areas of decision-making such as engineering design, and financial portfolio selection. This site presents a focused and structured process for optimization analysis, design of optimal strategy, and controlled process that includes validation, verification, and post-solution activities.

The effectiveness of the results of the application of any optimization technique is largely a function of the degree to which the model represents the system studied. Constraints limit the possible values for the decision variables in an optimization model. There are several types of constraints. The classes that implement them all inherit from the Constraint class. Each constraint also has a Name, which may again be generated automatically. After the solution of the model has been computed, the Value property returns the value of the constraint in the optimal solution. There are two types of constraints: linear and nonlinear (Bloom, 1956). Linear constraints express that a linear combination of the decision variables must lie within a certain range. Nonlinear constraints express that the value of some arbitrary function of the decision variables must lie within a certain range.

Optimization" comes from the same root as "optimal", which means best. When you optimize something, you are "making it best". But "best" can vary. If you're a football player, you might want to maximize your running yards, and also minimize your fumbles. Both maximizing and minimizing are types of optimization problems. In mathematics, computer science and operations research, mathematical optimization or mathematical programming, alternatively spelled optimization, is the selection of a best element (with regard to some criterion) from some set of available alternatives (Aderson, 1998).

Optimization is the process of finding solutions to the conditions that give the minimum or maximum value of a function, where the function represents the effort required or the desired benefit. The objective of optimization is to achieve the "best" design relative to a set of prioritized criteria or constraints. These include maximizing factors such as productivity, strength, reliability, longevity, efficiency, and utilization (Ted, 2012). Mathematical Optimization is a branch of applied mathematics which is useful and very fundamental in many different fields. Such as: Manufacturing, Production, Inventory control, Transportation, Scheduling, Networks, Finance, Engineering, Mechanics, Economics, Control engineering, Marketing and Policy Modeling.

Conceptual Clarification

The following concepts were clarified in this paper:

Model Classifications

Emmanuel and Jacek (1999) considered the following as some of the model classifications:

1. Physical Models: Physical models are the ones that look like the finished object they represent. Iconic models are exact or extremely similar replicas of the object being modeled
2. Schematic Models: Schematic models are models that provide pictorial representations of mathematical relationships. Plotting a line on a graph indicates a mathematical linear relationship between two variables. Two such lines can meet at one exact location on a graph to indicate the break-even point, for instance; Pie charts, bar charts, and histograms can all model some real situation, but really bear no physical resemblance to anything. Diagrams, drawings, and blueprints also are versions of schematic models.
3. Verbal Models: Verbal models are the use of words to represent some object that exists, in reality. Verbal models may range from a simple word presentation of scenery described in a book to a complex business decision problem (described in words and numbers).
4. Mathematical Models: Mathematical models are built using numbers and symbols that can be transformed into functions, equations, and formulas. They also can be used to build much more complex models such as matrices or linear programming models.

Need of Optimization Modeling Tools

According to Emmanuel and Jacek (1999) considered the following as some of the steps of modeling process:

1. Go through them repeating certain steps by steps many times.
2. Recognition and definition of the real problem is often the most difficult step. For such a problem, the modeler can formulate the objective function and the set of constraints so producing an optimization model.
3. The collection of data: Data collection, according to researchers the most demanding task which are difficulties at this stage are the lack of data.

Typologies of Mathematical Models

Emmanuel and Jacek (1999) considered the following as some of the typologies of mathematical models:

1. Descriptive Models: Descriptive models are used to describe something mathematically. Common statistical models in this category include the mean, median, mode, range, and standard deviation. Balance sheets, income statements, and financial ratios are also descriptive in nature.
2. Optimization Models: Optimization models are used to find an optimal solution to mathematical problem. The linear programming models are mathematical representations of constrained optimization problems. Knowledge of these models helps us to recognize problems that can be solved using linear programming.
3. Reductionist and Holistic Models: Reductionist models are based on the attempt to include as many details as possible into the model and to describe the behavior of a system as the net effect of all processes.
4. Internal and External Models: Internal models describe system response as a consequence of input using the mechanistic structure of the system, whereas external (or input/output, black-box, empirical) models are based on empirical relationships between the input and the output.
5. Dynamic and Static Models: This classification arises between models that do or do not vary with time. Static models are referred to as steady-state models. They model the equilibrium behavior of the system.
6. Deterministic and Stochastic Models: Deterministic models all future outcomes are known with precision by the present state and the future values of external variables (inputs) of the model. Stochastic models take into account the random influences of the temporal evolution of the system itself.
7. Linear and Nonlinear: These are mathematical models that are used to compose by variables, which are abstractions of quantities of interest in the described systems, and operators that act on these variables, which can be algebraic operators, functions, differential operators.
8. Lumped and Distributed Parameters: A situations were by the model is homogeneous the parameters are distributed and if the model is heterogeneous then the parameters are lumped.

Optimization Problems Solving Processing

Yinyu (2010) considered the following as some of the optimization problems solving process:

1. Unconstrained Optimization: is the entire space.
2. Linear Optimization: when both the objective and the constraint functions are linear/affine
3. Conic Linear Optimization: A situation when both the objective and the constraint functions are linear/affine, but variables in a convex cone.
4. Nonlinear Optimization: A process when the constraints contain general nonlinear functions.
5. Stochastic Optimization: Optimize the expected objective function with random parameters.

6. Fixed-Point Computation: Optimization of multiple agents with zero-sum objectives.

Steps in the Optimization Problem-Solving Process

(1) Draw a diagram based on the problem scenario, but show only the essentials aspects.
(2) Give the diagram symbols.
(3) Analyze and discus the diagram and relating the "knowns" to the "unknowns".
(4) Find the extreme values using the Calculus in terms of mathematical expression.

A mathematical optimization problem also consists of:

i. A set of variables that describe the state of the system.
ii. A set of constraints that determine the states that are allowable.
iii. External input parameters and data.
iv. An objective function that provides an assessment of how well the system is functioning.
v. The variables represent operating decisions that must be made.
vi. The constraints represent operating specifications.
vii. The goal is to determine the best operating state consistent with specifications.

Modeling Techniques of Mathematical Optimization

The following are some the techniques of the mathematical optimization:

a. Classical optimization techniques.
b. Linear programming.
c. Nonlinear programming
d. Geometric programming,
e. Dynamic programming.
f. Integer programming
g. Stochastic programming.
h. Evolutionary algorithms.

Reasons of using Mathematical model during teaching mathematics

The following are some of the reasons of teaching mathematical models:

1. Mathematical models help students to think critically and locally before solving a problem.
2. Mathematical models help students understand and explore the meaning of equations or functional relationships.
3. Mathematical modeling software such as Excel, on-line JAVA /Macromedia type programs make it relatively easy to create a learning environment in which introductory students can be interactively engaged in guided inquiry, heads-on and hands-on activities.
4. Quantitative results from mathematical models can easily be compared with observational data to identify a model's strengths and weaknesses.
5. Mathematical models are an important component of the final "complete model" of a system which is actually a collection of conceptual, physical, mathematical, visualization, and possibly statistical sub-models for the good achievement of students in mathematics

Methods of Teaching Mathematical Optimization

Teaching generally, is a scientific and cognitive process of imparting knowledge to the learners with well environmental arrangement which involves an organism mentally and/or physically in a set organized activities, and facilities in order to achieve instructional objective. In view of the above definition, the following are some of the methods of teaching mathematical optimization:

1. Problem-solving method: The problem-solving method is a student-centered method in which students learn about a course through the experience of solving an open-ended problem. Students are formed in small groups guided by a tutor or an expert towards discovering answers on their own rather than to simply provide the correct answer (Wikipedia, 2017 & Barrow, 1996).
2. Project based learning method: The project method is an experience centered strategy related to life situation. It is a strategy whereby students are given a real world situation which they analyze and present using their academic knowledge and creativity. Duch (2002) described project based learning as an instructional method that challenges students to learn how to learn, working cooperatively in groups to seek solutions to real world problem. Bas (2011) undertook a study on the effects of project-based learning on students' academic achievement and attitudes. The result of the research showed that project-based learning was more effective in the positive development of students' academic achievement level. The findings also revealed that students educated by project-based learning were more successful and had higher attitude levels towards the lesson than the students who were educated by the instruction based on student textbooks. The results of the study also indicated that the activities used in the treatment provided more opportunities for the students to get involved in the activities than the participants exposed to the usual classroom lecture method.
3. Demonstrating Method: Demonstrating is the process of teaching through examples or experiments. For example, a science teacher may teach an idea by performing an experiment for students. A demonstration may be used to prove a fact of mathematics models through a combination of visual evidence and associated reasoning. Demonstrations method helps to raise student interest and reinforce memory retention in mathematics because they provide connections between facts and real-world applications of those facts. Demonstration method has been shown to be effective with both large and small groups during teaching and learning mathematics optimization. Demonstration teaching method is always accomplished by telling or explaining some concepts of mathematics by the handling or manipulating of real things, equipment or materials or showing pictures. Dorgu in Titus and Babangida (2018) argued that demonstration teaching method is useful mostly in imparting psychomotor skills and lessons that require practical knowledge. The gains of using demonstration teaching method in teaching lies in the fact that it bridges the gap between theory and practice, enables students to become good observers and generate their interest; students see immediate progress as a result of a correct effort and it enables the teacher to teach manipulative and operational skills. Demonstration teaching method is an effective means of supplementing and clarifying the content of mathematics being taught. Demonstration teaching method is that type of method has been shown to be effective with both large and small groups in improving students' learning in Mathematics optimization.

4. Collaboration Method: this allows mathematics students to actively participate in the learning process by talking with each other and listening to other points of view. Collaboration establishes a personal connection between students and the topic of study and it helps students think in a less personally biased way. Teachers may employ collaboration to assess student's abilities to work as a team, leadership skills, or presentation abilities.

Implications

Mathematics teachers are closely scrutinizing the adopted curriculum for students of advanced intellect in mathematics. Having a strong knowledge of mathematical facts is rarely criticized unless it comes at the expense of having skills. In other words, some practical aspects with mathematical problem-solving tasks are requisite to be able to do them successfully in the future. In this situation, students must be provided with serious challenges that enable them to utilize skills. Without significant access to tasks, the potential by-product is that academically advanced mathematics students may be inclined to become bored; have negative affect, such as attitude, interest, and value about mathematics; and become disengaged and less persistent with excessive exercises and word problems (Chamberlin, 2002).

Conceptual Understanding of Algorithms and Authentically Challenging Tasks Are needed for both the mathematics teachers and students for teaching and learning (Hiebert, 1997). Hiebert stressed that this would appear to be an emphasis of all mathematics classrooms, although reality may prove otherwise. This may be a familiar tactic to teachers with poor content knowledge. When "why" questions are used, the true meaning of mathematics may surface. Consequently, if academically advanced mathematicians are to be challenged in mathematics, it is incumbent upon mathematics teachers in elementary grades to help students consider why certain procedures have been accepted by the mathematics community as the most efficient method available for teaching and learning.

Mathematics instructional materials may not suffice: Perhaps the teaching method that requires the least effort is to open the textbook and use the problems for the lesson. However, many textbooks may not have a satisfactory number of authentically challenging tasks for students of advanced academic capabilities, although some exceptions appear to exist (Chamberlin et al., 2009). These problem selections of instructional materials (textbooks) may not even have authentically challenging tasks for students in the general population. Mathematics textbooks may be a compilation of exercises and word problems, which are very helpful if the objective is to hone low-level skills.

CONCLUSIONS

Optimal instruments calibration technique based on the method of least squares and on correction with comparable argument and function errors is described. Expressions are derived for calculating linear regression coefficients and calibration errors. The process of calibration when the instrument signals are nonlinear functions of the measured parameter and when the measurement error is different at different points of the measurement range is discussed.

Since teaching and learning of mathematics are the scientific and cognitive process of imparting knowledge to the learners with well environmental arrangement which involves an organism mentally and/or physically in a set organized activities, and facilities in order to achieve instructional objective, the application of the teaching methods should use appropriately for the clearly understanding during teaching and learning of the mathematical optimization. The demonstration and problem solving methods may be used to prove a fact through a combination of visual evidence and associated reasoning. The project method on the other hand is an experience centered strategy related to life situation. It is a strategy whereby students are given a real world situation which they analyze and present using their academic knowledge and creativity in learning mathematics.

Suggestions

Based on the foregoing discussion, the following suggestions are made:

1. Good instrument should be used to solve optimization problem in mathematics.
2. Mathematical expression should be use when solving optimization problem in any field.
3. Mathematicians should consider models as a guide for practical expression and understanding in solving optimization problem.
4. Mathematics teachers and students should apply the application of problem solving teaching method to teach and learn mathematical optimization.

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